Norwegian University of
Science and Technology

CONCRETE NTRU SECURITY AND ADVANCES IN PRACTICAL LATTICE-BASED ELECTRONIC VOTING

Presentation for NaCl

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- We adapt the E-Voting scheme from Aranha et al (CCS 2023) using NTRU to reduce ciphertext size and enc/dec time
- We implement this scheme to obtain timings
- Our resulting scheme is on average faster and smaller in size than previous work, but with a larger proof of boundedness.

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The NTRU problem

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Let $q > 2$ be a prime, d be the ring dimension and $D_{\sigma_{NTRU}}$ be a distribution over R_q. Sample $(f, g) \leftarrow D_{\sigma_{NTRU}}$, reject if f is not invertible in R_q, let $h = g/f \in R_q$

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Search-NTRU: given h, recover any rotation $(X^i f, X^i g)$ of (f, g).

Decision-NTRU: given h, decide if h is computed as $h = g/f$, or if h is uniformly sampled from R_q.

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- f and g are rejected unless their 2-norm is below some bound

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3. If $\|f\|_2 > t \cdot \sqrt{d} \cdot \sigma_{NTRU}$ or $\|g\|_2 > t \cdot \sqrt{d} \cdot \sigma_{NTRU}$, restart.

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3. If $\|f\|_2 > t \cdot \sqrt{d} \cdot \sigma_{NTRU}$ or $\|g\|_2 > t \cdot \sqrt{d} \cdot \sigma_{NTRU}$, restart.
4. Return secret key $sk = f$, public key $pk = h := g/f$

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$Dec(c \in R_q, sk = f)$:

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- Revealing that the NTRU problems becomes much easier to solve when σ is small and $q >> d$
- This “point” is referred to as the *fatigue* point.
- This is done by figuring out at which point it is easier to discover a dense sublattice generated by the secret key using BKZ than to find a secret key in the reduced basis

Our NTRU Fatigue Experiments

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- Previous work by by Ducas and van Woerden found experimentally that the fatigue point was when $q \approx d^{2.484}$ for ternary case NTRU
- We repeat their experiments/estimates, adjusting for different values of std. variation σ when sampling secrets from a gaussian
- We estimate that the concrete fatigue point of NTRU can be described by: $q = 0.0058 \cdot \sigma^2 \cdot d^{2.484}$, which we also confirm experimentally.

Hardness Results

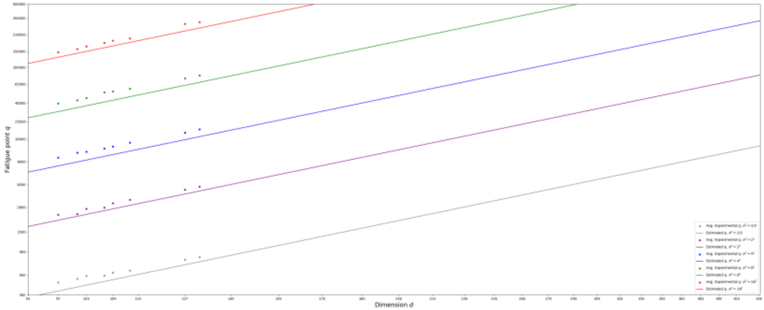

Fig. 2. Average experimental fatigue point q values plotted against estimated fatigue point using progressive BKZ with 8 tours on matrix NTRU instances with variance $\sigma^2 \in \{2/3, 4, 16, 64, 256\}$. The straight colored lines show the estimated values using the (modified) estimator from [DvW21]. The colored dots show the experimental results, where a DSD event has a 50% chance of triggering before an SKR event. The plot is scaled to $\log q$ and $\log d$.

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Therefore, we need algorithms for:

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- Distributed Decryption
- Mechanisms to verify that encryption, decryption and shuffling are computed honestly/correct

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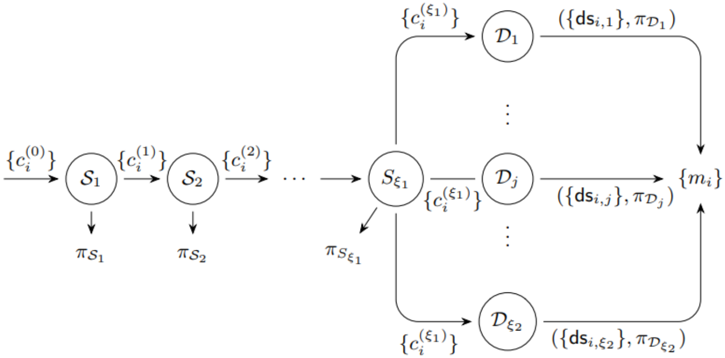
- **Ballot Casting:** Voters cast their ballot, their device encrypts it, along with a ballot proof
- **Ballot Counting:** Encrypted ballots are shuffled, a shuffle proof is created, the decryption servers receives ballots, verifies the shuffle proofs, computes decryption shares and proof of correct decryption, the decryption shares are then combined after decryption proofs are verified

Components of our E-Voting Scheme



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3. Sample a random permutation $\pi \leftarrow Perm[\tau]$
4. Return new set of ballots $\left\{\hat{c}_{(\pi(i))}\right\}_{i \in [\tau]}$

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- Create a proof that a batch of equations: $As_i = t_i$ for $i \in [l]$ is satisfied for a set of secret vectors s_i with ∞-norm bounded by ν

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- Return $ds_j = (\{ds_{ij}\}_{i\in\tau}, \pi_D)$

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Π_{LIN} produces a proof that a committed value v is a multiple of another commited value u with respect to a public scalar g.

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Compared to previous works we use exact proofs to get better parameters, though this leads to larger proof sizes.

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Decryption Correctness A ciphertext after the mix-net of ξ_1 shuffle servers is on the form

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So correct decryption would have to satisfy

$$p \cdot d \cdot t \cdot \sigma_{NTRU} \cdot (2\xi_1 \cdot \nu + 1/2)(1 + 2^{sec}) < \lfloor q/2 \rfloor$$

Parameters

Parameter	Explanation	Value
λ	Computational security parameter	128
d	Ring dimension	2048
q	Ciphertext and commitment modulus	$\approx 2^{59}$
sec	Statistical security parameter	40
p	Plaintext modulus	2
t	$KeyGen$ rejection parameter	1.058
ν	Infinity norm of encryption randomness	1
B_{Com}	Infinity norm of commitment randomness	1
ξ_1, ξ_2	Number of shuffle and decryption servers	4
σ_{NTRU}	Standard deviation for encryption secret key	7.12

Our Implementation

Build upon
https://github.com/dfaranha/lattice-voting-ctrsa21 and
https://github.com/dfaranha/lattice-verifiable-mixnet
by Diego Aranha

Size Comparison

Scheme	c_i	$[\![R_q]\!]$	π_{Shuf}	π_{Lin}	π_{Small}	π_{Bnd}
[7] [KB]	80	80/120	150	35	20	2
Our [KB]	15	30	63	18	22	22

Table 3: Ciphertext, commitment, and proof sizes per voter. Note that the two sizes in [7] reflect commitments to noise-drowning terms and ciphertexts, respectively.

Timing Comparison

Scheme	Com	Open	Enc	Dec	DDec
[7] [ms]	0.45	2.76	0.74	0.64	1.56
Our [ms]	0.17	0.80	0.20	0.21	0.45

Table 4: Ciphertext and commitment timings. Numbers were obtained averaging over 10^4 executions measured using the cycle counter available on the platform.

Questions?