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PRIO: PRIVATE, ROBUST, AND SCALABLE COMPUTATION OF AGGREGATE STATISTICS

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Contents

Key Contributions

Introduction and Motivation

Secret-Shared Non-Interactive Proofs (SNIPs)

Prio

More Building Blocks!

Fun Facts



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 Introduction of Secret-Shared Non-Interactive Proofs (SNIPs).



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- Introduction of Secret-Shared Non-Interactive Proofs (SNIPs).
- Presentation of affine-aggregatable encodings, unifying many data-encoding techniques for private aggregation.
- Demonstration of combining these encodings with SNIPs to ensure robustness and privacy in large-scale data collection.



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Overview



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 Manages to achieve: privacy and robustness against faulty and malicious clients



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Prio is also scalable



Overview

- Manages to achieve: privacy and robustness against faulty and malicious clients
- Prio is also scalable
- Achieves this with the use of a novel technique: secret-shared non-interactive proofs (SNIPs)





Machine Learning



Machine Learning

Health/fitness tracking



- Machine Learning
- Health/fitness tracking
- Web browsing data collection



- Machine Learning
- Health/fitness tracking
- Web browsing data collection
- Essentially in every scenario where aggregate data is valuable, but user privacy is just as important





 Prio has a minimal slowdown compared to non-private systems



- Prio has a minimal slowdown compared to non-private systems
- It also has a significant performance advantage over systems using conventional ZK approaches



Comparison

Туре	Prio	Prio (non-robust)	NIZK
Client-side cost	50x slowdown	N/A	50-100x vs. (compared to Prio)
Server-side cost	1-2x slowdown	5-15x slowdown	267x slowdown
Overall system	5.7x slowdown	N/A	N/A



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Experiment

Case: Privately collect responses to a survey with 434 true/false questions.

Results:

- Client: 26ms computation
- Servers: 2ms computation per submission



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 Cryptographic tools that allow a client to prove to a set of servers that a submitted value is correct and within expected parameters, without revealing the actual value.



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- Cryptographic tools that allow a client to prove to a set of servers that a submitted value is correct and within expected parameters, without revealing the actual value.
- Designed to work in a distributed setting where multiple servers collaboratively verify the correctness of client submissions.



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So what is the difference?

- SNIPs are tailored for efficiency in client/server settings.
- SNIPs are specifically designed for data aggregation settings, while NIZKs have a broader range of applications.
- SNIPs generally use a combination of polynomial identity tests and secret sharing, and are usually focused on information theoretic security.





Setup



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 Client evaluates Valid(x) on input x to know the value of every wire in the circuit



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Client Evaluation

- Client evaluates Valid(x) on input x to know the value of every wire in the circuit
- Client uses wires to construct polynomials f, g, h which encodes values on input and output wires of the M gates.





Polynomial Construction

• Let u_t, v_t be the input wires for the *t*-th multiplication gate



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- Define $h = f \cdot g$
- ▶ Then $\deg(f) \le M 1, \deg(f) \le M 1, \deg(h) \le 2M 2$
- Since $h(t) = f(t) \cdot g(t)$, then h(t) equals the output wire of the *t*-th gate.



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- Polynomial interpolation and multiplication to compute Valid(x)
- The client then splits the coefficients of h in s parts and sends the *i*-th share to the *i*-th server
- This way, only one honest server is needed to achieve information theoretic security (they each also only get x_i)



Consistency Checking

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- ► From this, the servers produce shares *f*_{*i*}, *g*_{*i*} without communicating with each other.
- If clients and servers all act honestly, then correctness is obvious



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- Since $\hat{h}(t_0) \neq h(t_0) = f(t_0) \cdot g(t_0) = \hat{f}(t_0) \cdot \hat{g}(t_0)$, then $\hat{h} \neq \hat{f} \cdot \hat{g}$ for the least t_0 s.t. $\hat{h}(t_0) \neq h(t_0)$





Polynomial Identity Test

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- Servers publish σ_i and ensure $\sum_i \sigma_i = 0$, if not reject



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- In this setting, the client generates a, b, c and splits into shares a_i, b_i, c_i for each of the servers.
- This saves computation time/resources.





Beavers MPC Protocol

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- $d_i = f_i(\tau) a_i, e_i = g_i(\tau) b_i$ where τ is the last multiplication gate of the circuit *C*.
- Each server broadcasts d_i, e_i
- Each server calculates $\rho_i = de/s + db_i + ea_i + c_i$.



Beaver MPC Protocol Correctness

$$\sum_{i} \rho_{i} = \sum_{i} (de/s + db_{i} + ea_{i} + c_{i})$$

= $de + db + ea + c$
= $(f(\tau) - a)(g(\tau) - b) + (f(\tau) - a)b + (g(\tau) - b)a + c$
= $f(\tau)g(\tau) - ag(\tau) + ag(\tau) - ab + c$
= $f(\tau)g(\tau) - ab + c$
= $f(\tau)g(\tau)$ = $h(\tau)$



Output Verification



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Servers publish output shares after the circuit



Output Verification

- Servers publish output shares after the circuit
- Sum up shares to confirm Valid(x) = 1





Security

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- Correctness follows construction.
- A malicious client must cheat the polynomial identity test with probability $(2M 2)/|\mathbb{F}|$.
- Completeness nor soundness holds in the presence of malicious servers.
- Malicious servers can mount selective DoS attacks against clients
- As long as at least one server is honest, dishonest servers learn nothing about the clients data.





Efficiency

Server-to-server communication cost grows neither with complexity of the verification circuit nor with the size of x.



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- Computation cost for each server is not much more than to evaluate the *Valid* circuit.
- Client-to-server communication cost grows linearly with the size of the *Valid* circuit.
- The authors note an interesting challenge to try to reduce the communication cost without needing expensive asymm. cryptography.



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We have s servers.



Upload



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Each client *i* splits its private value x_i into *s* shares



Upload

- Each client *i* splits its private value x_i into *s* shares
- ▶ Then sends this share $[x_i]_j$, $j \in [s]$ to each corresponding server j.





Aggregate



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Each server *j* holds an accumulator value $A_j \in \mathbb{F}_p$


Aggregate

- Each server j holds an accumulator value $A_j \in \mathbb{F}_p$
- ▶ And updates this $A_j \leftarrow A_j + [x_i]_j \in \mathbb{F}_p$ each time it receives a new value.



Publish



Publish

Once the servers have received all clients shares, they publish A_j.



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- Once the servers have received all clients shares, they publish A_j.
- Computing $\sum_j A_j \in \mathbb{F}_p$ yields $\sum_i x_i$.





Our Setting



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- Each client *i* holds a value $x_i \in D$, where *D* is some set of data values.
- The servers holds an aggregation function $f: D^n \to A$.
- The servers goal is to evaluate $f(x_1, \ldots, x_n)$ without learning $x_i \forall i$.





What do AFEs do?

► Gives an efficient way to encode data values x_i s.t. it is possible to compute f(x₁,...,x_n) given only the sum of the encodings of x₁,...,x_n.



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- ▶ Valid(y): returns true iff $y \in \mathbb{F}^k$ is a valid encoding of some data item in D.
- Decode(σ): Takes $\sigma = \sum_{i=1}^{n} Trunc_{k'}(\mathsf{Encode}(x_i)) \in \mathbb{F}^k$ and outputs $f(x_1, \ldots, x_n)$



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Implementation



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The prototype is only 5700 lines of Go and 620 lines of C (for FLINT)



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 The prototype is only 5700 lines of Go and 620 lines of C (for FLINT)

Code is available on https://crypto.stanford.edu/prio/.



Questions?

